

MAS115: Mathematical Investigation Skills

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Marcus Fabius Quintilian

(AD35–AD100)

We should not write so that it is possible for our readers to understand us, but so that it is impossible for them to misunderstand us.

De Institutione Oratoria, Book VIII, 2, 24

Introduction

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We will look at how to

- typeset mathematical documents (Weeks 1–6);
- create mathematical webpages (Weeks 8–11);
- write clear and concise content to go in them (throughout the year!).

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- be able to make create impressive mathematical webpages and reports.

Am I being clear?

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When writing, there's no-one to ask questions.

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Change the symbols for words: does what you have make sense?

Example.

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which is reads very well.

Example.

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Activity. Introduce yourself to the person next to you (or talk in a group of three). Write out a joint solution, trying to think of all the subtleties you can to improve the previous effort.

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Better! This is hopefully close to what you'd submit as a handwritten solution.

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Notice the use of the ' \approx ' sign!

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We'll practice typing solutions like the second one in the computer labs.

Using common symbols

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$$\begin{aligned}x^2 - 2 &= 0 \\ \therefore x^2 &= 2 \\ \therefore x &= \pm\sqrt{2}.\end{aligned}$$

These symbols are not used much (if at all) in typewritten mathematics.

Dos and don'ts of presentation.

Do

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*Now, if $x > 0$ then $x^3 + x > 0$. Thus, the function
 $f(x) = x^3 + x$ is positive for all positive real
numbers x .*

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The function $f : [0, 10] \rightarrow \mathbb{R}$ given by $f(x) = x^2$ takes its maximum value when x is maximum, namely at $x = 10$.

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People sometimes use vague arrows such as \rightsquigarrow , but what does this actually mean?

Summary

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So here's some advice for any students having to resit, or for those who passed to bear in mind for the future: treat mathematical presentation as key! Be strict on yourself with your thinking and take notice of how arguments are presented by lecturers in classes and in solutions. If there is one thing that can greatly improve ability (and hence boost marks), this is it!

About Computer Lab 1

Presentation Computer Lab 1 will introduce you to L^AT_EX.

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Presentation Computer Lab 1 will introduce you to \LaTeX . I hope you enjoy learning how your lecturers create their course materials.