MAS115: Mathematical Investigation Skills

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Marcus Fabius Quintilian (AD35–AD100)

We should not write so that it is possible for our readers to understand us, but so that it is impossible for them to misunderstand us.

De Institutione Oratoria, Book VIII, 2, 24

Introduction

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We will look at how to

- typeset mathematical documents (Weeks 1–6);
- create mathematical webpages (Weeks 8–11);
- write clear and concise content to go in them (throughout the year!).

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- be more fluent in your mathematical writing;
- be able to make create impressive mathematical webpages and reports.

Am I being clear?

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When writing, there's no-one to ask questions.

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Change the symbols for words: does what you have make sense?

Example.

$\forall x$, $e^x > 0$.

If $e^x(x^2 - 4) > 0$ *then* $x^2 - 4 > 0$.

$\therefore$ $(x + 2)(x - 2) > 0$, *so* $x < -2$ *or* $x > 2$.

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For all x, e^x is greater than 0.

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$r^2 = A/\pi$

$\sqrt{A/\pi}$ $\sqrt{10/\pi}$

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The square-root of A/π

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which is dreadful!

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Activity. For the bad solution above, try to work out what question was asked,

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Pause the video now and have a go!

That example was a bad attempt at answering the following.

> *'Write down an expression for the radius* r *of a circle of area* A*. Use it to find the radius when the area is 10 cm*^{2}*.'*

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Better! This is hopefully close to what you'd submit as a handwritten solution.

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Notice the use of the '$\approx$' sign!

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The area A for a circle with radius r is given by $A = \pi r^2$. Thus $r^2 = \frac{A}{\pi}$ and so $r = \sqrt{A/\pi}$, since $r > 0$. When $A = 10$ this gives $r = \sqrt{10/\pi}$.

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We'll practice typing solutions like the second one in the computer labs.

Dos and don’ts of presentation.

Do

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in which case use a comma,

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in which case use a comma, or nothing at all.

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> *Now,* $x > 0 \implies x^3 + x > 0$*.* $\therefore f(x) = x^3 + x > 0$ *for all* $x \in \mathbb{R}^+$*.*

as much as

> *Now, if* $x > 0$ *then* $x^3 + x > 0$*. Thus, the function* $f(x) = x^3 + x$ *is positive for all positive real numbers* x*.*

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can be re-written as

> *The function $f : [0, 10] \to \mathbb{R}$ given by $f(x) = x^2$ takes its maximum value when x is maximum, namely at $x = 10$.*

Don’t

Don’t invent your own symbols.

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People sometimes use vague arrows such as $\rightsquigarrow$, but what does this actually mean?

Summary

Good mathematical presentation is important. It helps to create clear, readable solutions and helps you to think clearly too.

About Computer Lab 1

Presentation Computer Lab 1 will introduce you to LaTeX.

Important! Before the lab (do it now!), go to `http://www.overleaf.com` and register for an account so that you are ready to go when the lab starts.

I hope you enjoy learning how your lecturers create their course materials.