Lecture 4 Activity

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1 Gödels Incompleteness Theorem

”A man says that he is lying. Is what he say’s true or false.”
Eubulides, 4th century BC

In the late 19th and early 20th centuries, concerns developed that mathematics often seen as the pinnacle of pure reasoning was built on shakey foundations. Paradoxes were found at the core of the logic used to discuss collections (or sets). Bertrand Russell and Alfred North Whitehead, amongst others, embarked on a mission to codify the whole of mathematics by axiomatising the theory of set's; using formal logic as a basis. From there they, defined developed and reproved in unambiguously rigourous ways a huge amount of mathematics, published in the three-volume work Principia Mathematica. Ultimately, they hoped and believed, any true mathematical statement could be proven within their framework!

in 1931, fresh from completing his doctorate, kurt gödel published a relatively short paper which destroyed the ambitions of russell and whitehad. Not only did he manage to prove: using an ingenious reworking of the liar paradox this sentence is false, that there were true mathematical statements that would never be provable in the framework of Principia Mathematica, but also that any attempts by Russell and Whitehead to patch up their system would fail. In other words:- any axiomatic codification of mathematics will always by incomplete.

For those who are interested, the short book ‘gödel’s proof’ by Nagel and Nemwan is an easily readable and non-technical overview of the area, and the page

http://www.miskatonic.org/godel.html

may also whet the appetite