MAS115 PRESENTATION LAB 3

As usual, go to Overleaf and start a new project called 'MAS115 Week 3'. Include the usual preamble (using documentclass, begin and end document commands etc).

> *I often borrow the preamble from a previous .tex file. For example, you could use the one my Week 2 lab attempt from the course website, or simply start from the Overleaf default.*

Add the title 'MAS115: Week 3 Experiments'. Include the AMS packages with the command `\usepackage{amsmath,amsthm,amssymb}` in the preamble. (Look back at the Week 1 and 2 documents if you're stuck!)

> *Those who prefer using MiKTeX or MacTex on their own computer may like to download and install* TeXmaker*, which is a text-editor that can be used as an alternative to TeXworks/TeXshop. Some people prefer TeXmaker to TeXworks because it has lots of useful extra buttons, e.g. lists of symbols.*

Typesetting practice

(1) Start a section called 'Typesetting practice'. Begin an enumerate environment and typeset the following as the first `\item`.

> The formula for the addition of velocities in relativistic mechanics is
> $$f(u,v) = \frac{u+v}{1+\frac{uv}{c^2}}.$$

Once you've typed it in, see what happens as you move past the brackets in the code with the cursor. (This is a useful feature for checking brackets match up, particularly when they're 'nested' like here.)

(2) By looking on the Overleaf List of Greek letters and math symbols, typset the following as the second `\item` on your list.

> Overleaf has a list of symbols, making it easy to typeset things like
> $$\Xi^{\Psi\Omega} = \frac{\theta}{\gamma} \Longleftrightarrow \chi = \upsilon(\rho).$$

> *Those using TeXmaker can click on the small* $\boxed{\lambda}$ *and* $\boxed{\Rightarrow}$ *buttons on the left-hand side of the screen to find the commands.*

(3) Try the command `$\underbrace{x+y}_{z}$`, and then adapt to see you can figure out how to typeset the below.

As you learn more commands, even things like the below become easy.

$$\underbrace{\partial \dots \partial}_{n \text{ times}}$$

Hint: '...' is best achieved with `\ldots`, and `\text{}` allows text in math-mode and should help with the 'n times'.

Theorems

Start a new section called 'The square-root of 2'. Here we're going to experiment further with environments.

(4) Start with the text

> Here, we're going to investigate a solution of the equation
> $$x^2 = 2.$$

(5) Try using `\begin{equation}` and `\end{equation}` rather than double-dollars around the equation $x^2 = 2$. You should find that the *equation* environment numbers the displayed equation. If you want to put the equation numbers on the right (respectively on the left) add `reqno` (respectively `leqno`) to your document class command options, as in

```
\documentclass[11pt,reqno]{...}.
```

(6) We want to refer to the equation we've just created later. To make this easy to do, put `\label{eq:root 2}` after `\begin{equation}`.

(7) Now type

```
The positive solution to equation \ref{eq:root 2} is
denoted $\sqrt{2}$.
```

and process the file. Does it work? If you see '??' on your PDF, run it again. (LaTeX has to run twice to get autoreferencing to work, but usually it does this automatically.)

Now let's make our first theorem. Add the following to the preamble.

```
\newtheorem{thm}{Theorem}[section]
```

This will create a new environment called 'thm' which we'll use shortly. In the body, write

```
\begin{thm}
The real number $\sqrt{2}$ is irrational.
\end{thm}
```

Process the file, and look at the output. Underneath the theorem, write

```
\begin{proof}
This is a famous proof by contradiction.
\end{proof}
```

The proof that $\sqrt{2}$ is irrational relies on the fact that any rational number can be written in the form $\frac{a}{b}$ with a and b coprime integers. Let's include this as a lemma[1].

To do this, first add the following in the preamble just after the `\newtheorem{thm}` command.

```
\theoremstyle{plain}
\newtheorem{lem}[thm]{Lemma}
```

This creates a new environment called 'lem'. Using this, put a lemma before the statement of the theorem that looks like the following.

> **Lemma 0.1.** *Any rational number can be written in the form $\frac{a}{b}$ with a and b coprime integers.*
>
> *Proof.* Cancel if necessary. □

When you process the file, notice how the numbering for the theorem automatically changes.

Add the commands

```
\theoremstyle{definition}
\newtheorem{defn}[thm]{Definition}
```

in the preamble, after the `\newtheorem{lem}` command. Now change the line where you defined $\sqrt{2}$ into a definition using `\begin{defn}` and `\end{defn}` to make

> **Definition 0.2.** The positive solution to equation (1) is denoted $\sqrt{2}$.

Again, the numbering will automatically change.

Experiment with labelling and referencing your definition, lemma and theorem which is done in the same way as for equations.

You can read more about how the `\newtheorem` command works on the webpage `http://en.wikibooks.org/wiki/LaTeX/Theorems`.

[1] A *lemma* is a short, usually uninteresting result in its own right which is used as an intermediate step towards a proposition or theorem. In order of importance, theorems come above propositions, which come above lemmas.

Homework

Create a document with title 'MAS115: Homework 3' and your name on as author. Using the work you started in this sheet, write a good account of the square root of 2 and its irrationality by adding proper proofs of the lemma and theorem.

More specifically, you need to include a more detailed proof of why any rational number can be written as $\frac{a}{b}$ in such a way that a and b are coprime, and you also need a full proof that $\sqrt{2}$ is irrational.

- Write these proofs in your own words! You may be able to find versions of the proofs in your notes for other courses or on the web, so the idea is to first understand them, and then write your own accounts.
- Remember that rational numbers are defined to be those of the form $\frac{a}{b}$ for integers a and b with $b \neq 0$, so the key is proving that they can be chosen to be *coprime*.
- Your document should read well from start to finish, so take some care in how it's presented.
- Include a section title and introductory paragraph.
- Below you can find the relevant section from my attempt at this week's sheet which should give you a starting point for the layout for your homework.

Upload the homework before next week's computer class, as usual.

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1. The square-root of 2 (my lab attempt!)

Here, we're going to investigate a solution of the equation

$$x^2 = 2. \tag{1}$$

Definition 1.1. The positive solution to equation (1) is denoted $\sqrt{2}$.

Lemma 1.2. *Any rational number can be written in the form $\frac{a}{b}$, where a and b are coprime integers.*

Proof. Cancel if necessary. □

Theorem 1.3. *The real number $\sqrt{2}$ is irrational.*

Proof. This is a famous proof by contradiction. □