

# MAS115 R programming, Homework Solutions 6

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## 1 Calculation of population variance

- (a) Use the function as given, with argument `1:6` or `c(1,2,3,4,5,6)` or similar.

```
# Find the 'empirical' (or 'population' or 'uncorrected') variance of a vector
EmpVar <- function(x)
{
  n <- length(x) # Find the length of x
  return (sum(x^2) - n * mean(x)^2) / n) # Calculate and return empirical variance
}
> EmpVar(1:6)
[1] 2.916667
```

- (b) There are many ways to add this functionality. With only two options, a logical or boolean argument is probably the easiest. A character-valued argument is possible; R has some built-in tools to help with more complicated cases—see e.g. the help page for `charmatch`.

```
# Find either the empirical OR the usual sample variance of a vector
EitherVar <- function(x, SampleVar=FALSE)
{
  if(SampleVar)
  {
    v <- var(x)
  } else
  {
    n <- length(x) # Find the length of x
    v <- (sum(x^2) - n * mean(x)^2) / n
  }
  return(v)
}
> EitherVar(1:6)
[1] 2.916667
> EitherVar(1:6,SampleVar=TRUE)
[1] 3.5
> EitherVar(1:6,TRUE)
[1] 3.5
```

- (c) In theory, the value should be unaffected by adding a constant to all elements of the vector. See the following example (note that the inner parentheses are not necessary, but help clarify what is being passed to the function).

```
> EmpVar((1:6)+10^6)
[1] 2.916667
```

When adding larger constants i.e. larger values of  $m$ , the two terms involved in the subtraction can become so large compared with the difference that numerical accuracy is completely lost. For example:

```
> EmpVar((1:6)+10^7)
[1] 2.916667
> EmpVar((1:6)+1*10^8)
[1] 2.666667
> EmpVar((1:6)+2*10^8)
[1] -5.333333
```

- (d) To fix this, we could calculate the empirical variance without forming such large quantities along the way.

```
EmpVar2 <- function(x)
{
  n <- length(x) # Find the length of x
  mu <- mean(x) # find the mean of x
  return (sum((x-mu)^2) / n) # Calculate and return empirical variance
}
> EmpVar2(1:6+10^9)
[1] 2.916667
```

Alternatives would be to pre-process  $\mathbf{x}$ , by shifting to more ‘reasonable’ values without affecting the correct answer, or to use the built-in `var` to do the main work and then adjust the answer afterwards.

## 2 Newton-Raphson Algorithm

My Newton-Raphson code is as follows - note again that I have added useful comments with the `#` key so the reader know what the functions does. I have also added some lines which write to the screen while it is running just to make it a little easier to understand.

```
##### Newton-Raphson #####
# This code will implement Newton-Raphson method

# Define function to solve = 0
f1.func <- function(x) {
  x^2 - 2
}

# Define derivative of function
f1deriv.func <- function(x) {
  2*x
}

# Performs Newton-Raphson algorithm to solve equation funct = 0
# Input: x = starting point, funct = function to find zeros of,
#       deriv = function to calculate derivative,
#       maxit = maximum iteration number, tol = when to stop
# Output: x = estimate, iter = number iterations, converged = whether converged
```

```

myNR.func <- function(x, funct, deriv, maxit = 100, tol = 0.000001) {
  eps <- Inf
  iteration <- 0
  while((iteration < maxit) && (abs(eps) > tol)) {
    iteration <- iteration + 1
    f <- funct(x)
    df <- deriv(x)
    eps <- f/df
    x <- x - eps
    cat("Iteration = ", iteration, "Root estimate = ", x, "\n")
  }
  return(list(x = x, iter = iteration, converged = (iteration < maxit)))
}

```

When I run this code I find that it converges extremely quickly to  $\sqrt{2}$  as can be seen with

```

> myest <- myNR.func(2,f1.func,f1deriv.func)
Iteration = 1 Root estimate = 1.5
Iteration = 2 Root estimate = 1.416667
Iteration = 3 Root estimate = 1.414216
Iteration = 4 Root estimate = 1.414214
Iteration = 5 Root estimate = 1.414214
> myest
$x
[1] 1.414214
$iter
[1] 5
$converged
[1] TRUE

```

### Solving $\cos(x) = x$

To find the solution to  $\cos(x) = x$  we just need to change the function arguments passed to the main function. We want to find the root of

$$\begin{aligned}
 f(x) &= \cos(x) - x \\
 \Rightarrow f'(x) &= -\sin(x) - 1
 \end{aligned}$$

so we define functions

```

cosminusx <- function(x) {
  cos(x) - x
}

cosderiv <- function(x) {
  -sin(x) - 1
}

```

Running this code we find the answer to be  $x \approx 0.739$ :

```

> myest <- myNR.func(1,cosminusx,cosderiv)

```

```
Iteration = 1 Root estimate = 0.7503639
Iteration = 2 Root estimate = 0.7391129
Iteration = 3 Root estimate = 0.7390851
Iteration = 4 Root estimate = 0.7390851
```

```
> myest
```

```
$x
```

```
[1] 0.7390851
```

```
$iter
```

```
[1] 4
```

```
$converged
```

```
[1] TRUE
```