

Week 9: Exercise Solutions

Exercise 9.1.

The approximation gives the following.

$$\int_0^1 x^2 dx \approx 1/2(1/4)^2 + 1/2(3/4)^2 = 1/32 + 9/32 = 10/32 = 5/16 = 0.3125.$$

The actual answer is

$$\int_0^1 x^2 dx = \left[\frac{1}{3}x^3 \right]_0^1 = 1/3 \approx 0.33333.$$

Exercise 9.2.

```
def trapezoidal_approximation(f, N, a, b):  
    """  
    Calculate an approximation to  $\int_a^b f(x)$   
    with  $N$  trapeziums  
    """  
    approximation = 0  
    for i in range(N):  
        approximation += (b - a)/(2*N) * (f(a + (i)*(b - a)/N) +  
                                           f(a + (i+1)*(b - a)/N))  
    return approximation
```

Exercise 9.3.

The function is the factorial function. It works because the factorial function satisfies the following two equations, which suffice to calculate it for any natural number:

$$0! = 1 \quad \text{and} \quad n! = n \cdot (n - 1)! \quad \text{for } n \geq 1.$$