MAS115 R programming
Challenge: beds in a hospital

This problem is hard but I think it's quite fun. If you want to have a go at it to test what you've learnt over the past semester and a half then please feel free to do so. I will release solutions later for you to check your efforts.

The problem A hospital ward has 8 beds and we are worried about whether it is fit for purpose or will become too busy. Let us suppose that:

- The number of patients arriving each morning is uniformly distributed between 0 and 5 inclusive.
- The number of days spent in hospital for each patient is also uniformly distributed between 1 and 3 days inclusive. When their stay is complete, they are discharged in the evening. If they only stay a single day then they will leave that evening.

If all 8 beds are free initially, what is the expected number of days before there are more patients than beds?

Pseudo-Code To solve this problem we are going to use the following idea. Firstly, define W to be the number of days before the first patient arrives to find no available beds. The question has asked us to give $E(W)$. To do this we want to:

- Generate lots of potential $W_1, \ldots, W_n$ from the distribution of W,
- Estimate $E(W)$ by $\bar{W} = \frac{1}{n}\sum_{i=1}^{n} W_i$.

Even creating a single W_i still looks quite complicated though so lets start off by just considering what will happen on any particular day. Let us define

B	:	number of beds available at beginning of day
N	:	number of arrivals at beginning of day
F	:	vector of length 3 with F_i being number discharged on i^{th} day in future

Note: F_1 will be the number of people discharged that evening, F_2 tomorrow evening and F_3 the evening after.

A single day Now we can find out what happens on day k:

1. Determine how many patients arrive
   - Sample N from discrete uniform distribution on $\{0, 1, 2, 3, 4, 5\}$
   - If $N > B$, there are more patients than beds; report $W_i = k - 1$ and stop.
2. If $N > 0$ then generate the length of stay for these N arrivals and determine when they leave: For $l = 1, \ldots, N$:

   - Sample the length of stay S_l for arrival l from the discrete uniform distribution on $\{1, 2, 3\}$.
   - Set $F_{S_l} \leftarrow F_{S_l} + 1$.

3. Determine number of beds available for next day after discharging:
   - Set $B \leftarrow B + F_1 - N$.

4. Modify vector F to find out numbers discharging for tomorrow:
   - $F_j \leftarrow F_{j+1}$ for $j = 1, 2$
   - $F_3 \leftarrow 0$ for $j = 3$

Putting this all together We can now put this all together into one single piece of code to work out a single W_i:

1. Initialisation:
   - Set $B_1 \leftarrow 8$, $F_j \leftarrow 0$ for $j = 1, 2, 3$, and $k \leftarrow 1$
2. Consider day k and see if beds exceeded
3. If not exceeded, set $k \leftarrow k + 1$ and return to step 2.

Tasks

- Create variables B, k and F as described above with some initialising values of your choosing.
- Write code that implements the happenings in a single day.
- Incorporate the code for your single day into a loop (you decide which is best) which will give you an individual value W_i.
- Embed this code into another loop (again you decide which is best) to create 1000 values for W_i stored in a vector `W`.
- Find the mean of `W`.

Notes:

You need to be very careful making sure that your code for a single day will work if there are no arrivals that day. Using `for` *loops can be a bit tricky if for example for you have something like:*

```
N <- 0 # Suppose we wish to sample N values and have chosen N = 0
S <- rnorm(N)
for(i in 1:N) {
  print(S[i])
}
```

The aim of this might be to print out the values we have sampled but because the **R** command `1:0` will create the vector $(1, 0)$ it isn't what we get. You may want to use an `if` statement so that you only have to update F if the number of arrivals is greater than 0 as we have suggested in the pseudo-code.

Solution You should find that the $E(W) \approx 17.7$. If you plot a histogram of your results they should look something like Figure 1.

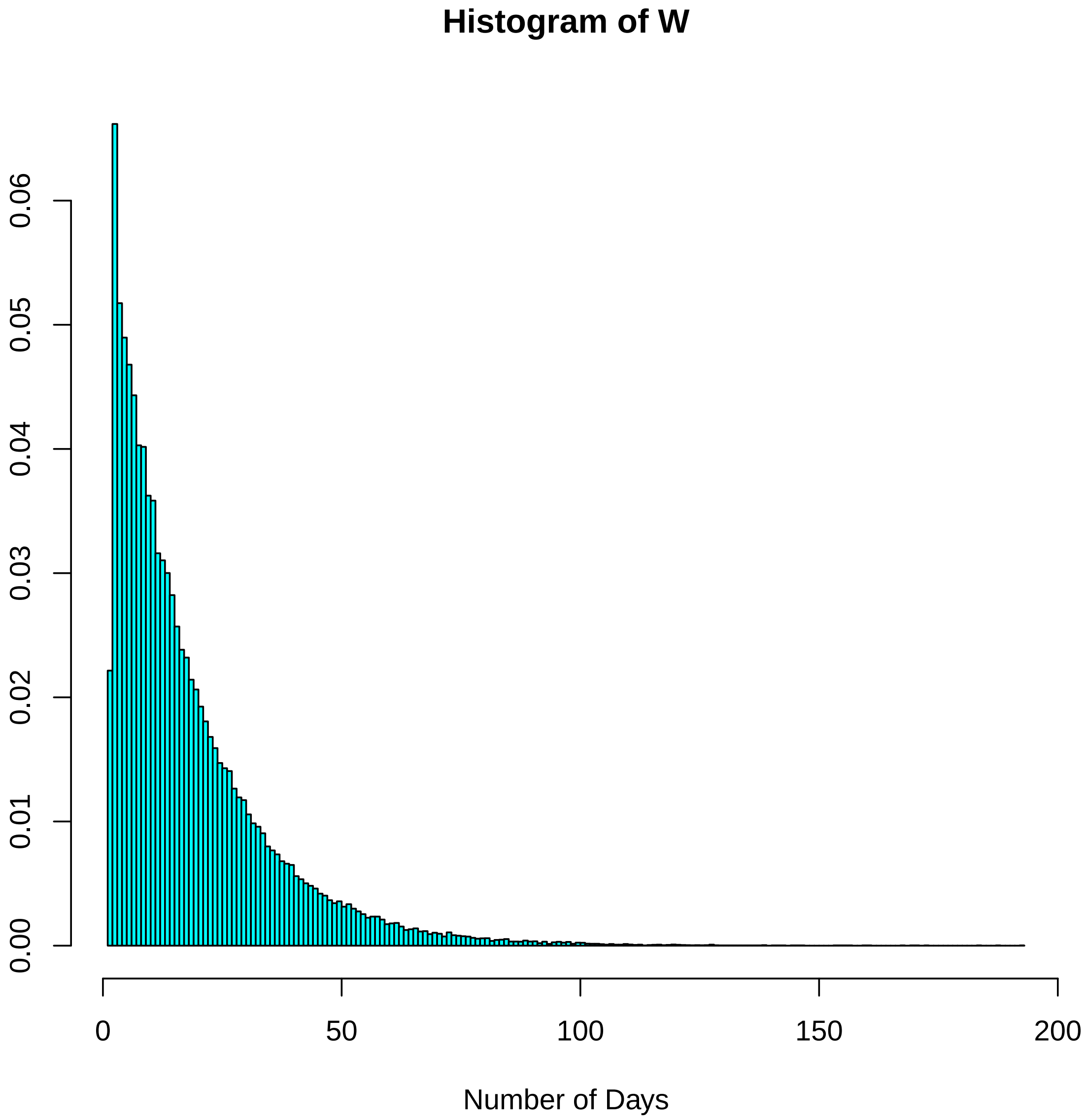

Figure 1: Histogram of the number of days that have passed before the hospital gets too full.