# Calculus examples

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## 1 Differentiation

Let $f(x) = \frac{e^x}{1-x}$. Then

$$f'(x) = \frac{(1-x)e^x - e^x.(-1)}{(1-x)^2} = \frac{e^x(2-x)}{(1-x)^2}.$$

## 2 Integration

Because the derivative of $\sin x$ is $\cos x$, it follows that

$$\int_0^\pi \cos x \; dx = [\sin x]_0^\pi = \sin \pi - \sin 0 = 0.$$

## 3 Formulae

The three main differentiation formulae are the product rule,

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx},$$

the quotient rule,

$$\frac{d}{dx}(u/v) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2},$$

and the chain rule,

$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$$

(where $y = y(u)$ is a function of $u$, and $u = u(x)$ is a function of $x$).

The rule for integration by parts is given by the formula

$$\int u\frac{dv}{dx} \; dx = uv - \int v\frac{du}{dx} \; dx,$$

where $u$ and $v$ are both functions of $x$.